Tunable Quasi-Static Suppression of Inertial and Passive Weak-Field Response of a Macroscopic Rigid Body via Vector Potential Coupling

Han, Jae Jun

Individual researcher

tobloqibs@icloud.com

A macroscopic body coupled to an internal electromagnetic mode can exhibit a frequency-dependent response coefficient, but interpreting that coefficient as inertia or passive gravitational mass requires dynamical and covariant matching[1, 2, 3, 4]. We analyse the proposed toroidal flux-gradient/LC architecture using the distinct quadratic actions specified here. Eliminating the physical LC charge gives a positive quasi-static inertia correction $+C_eG^2$. The independently postulated cyclic-channel action yields a negative kinetic Schur term; on its balanced branch, positive-definite kinetic energy imposes $M_{\mathrm{eff}}(0) > M/2$. We prove a stronger uniform-band existence theorem: for any fixed subresonant interval and arbitrarily small positive ϵ, a non-balanced positive-energy family has $0 < M_{\mathrm{eff}}(\omega) \leq \epsilon M$ throughout the interval; its kinetic condition number diverges toward the excluded zero endpoint. We prove that rigid co-translation of all internal magnetic sources and flux loops gives zero global-translation flux gradient; nonzero gradients require relative motion or an external reference. A covariant monopole completion gives equal inertial and weak-field passive coefficients in the stipulated translation sector, without establishing suppressed weight for a complete device[5]. The resulting inequalities and source-level response laws are analytic checks on the original simultaneous-suppression target, not evidence of an experimentally realized mass-null apparatus.

Introduction

A rigid body's centre-of-mass (COM) inertia differs conceptually from a frequency-dependent response inferred for a coordinate after internal modes have been eliminated. Mechanical resonators and circuits can display effective coefficients and dispersive susceptibilities without altering the conserved energy or proper rest mass of their complete physical implementation[1, 2, 3]. The distinction matters for a kilogram-scale apparatus in which currents, stresses, the drive electronics and the test frame all carry momentum and energy. We study a toroidal electromagnetic coupling to an internal LC mode; Maxwell–circuit matching, apparatus parameters and proposed calibrations are detailed in the Supplementary Information.

The initial task is to establish which coefficient follows from the stated action before assigning it an inertial, passive-gravitational or active-source interpretation. Here we use the explicit quadratic one-channel reduction, evaluate its zero-frequency response, and make its stability and SI-unit conditions explicit. The principal result is conditional on the declared reduced action; no experimental observation of suppressed mass is assumed.

Two different couplings

For a position-coupled oscillator, consider

$$L_{\mathrm{pos}} = \tfrac{1}{2}M\dot{X}^2 + \tfrac{1}{2}\mu\dot{q}^2 - \tfrac{1}{2}\mu\Omega^2 q^2 + gXq + FX. \tag{1}$$

With F an externally prescribed generalized force, correct Euler–Lagrange variation gives $M\ddot{X} - gq = F$ and $\mu\ddot{q} + \mu\Omega^2 q - gX = 0$. Thus

$$F(\omega) = \left[-M\omega^2 - \frac{g^2}{\mu(\Omega^2 - \omega^2)}\right] X(\omega). \tag{2}$$

The first correction is a static stiffness term, not a mass. Expansion at $|\omega| \ll \Omega$ gives

$$F(\omega) = \left[-\frac{g^2}{\mu\Omega^2} - \omega^2\left(M + \frac{g^2}{\mu\Omega^4}\right) + O(\omega^4)\right] X(\omega). \tag{3}$$

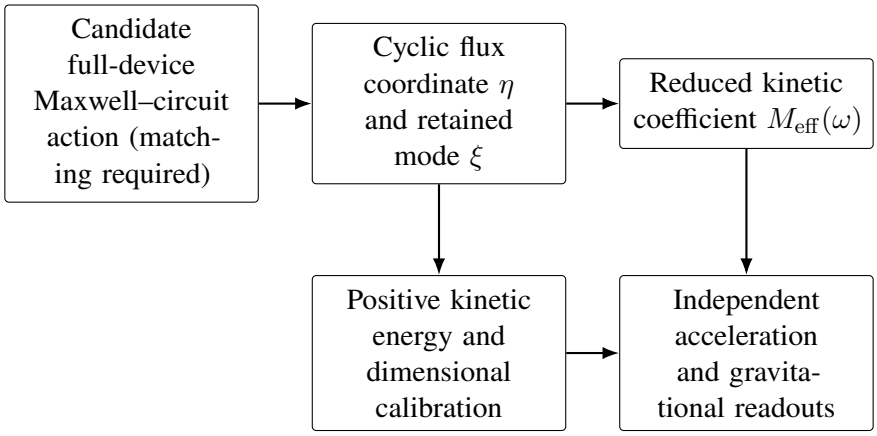


Figure 1: **Logical structure of the proposed electromechanical test.** The zero-frequency coefficient of a reduced translation coordinate is obtained from the matched quadratic action. Kinetic positivity, physical dimensions and independent gravitational coupling must then be checked before identifying it with an observable mass of the complete device. The diagram is schematic and does not represent experimental data.

Consequently, after separating the static stiffness, the *finite* inertial correction is $+g^2/(\mu\Omega^4)$, not $-g^2/(\mu\Omega^2)$. Moreover, the original potential $V = \mu\Omega^2 q^2/2 - gXq$ has an indefinite Hessian for $g \neq 0$. A supplementary restoring term $KX^2/2$ stabilizes the potential only for $K \geq g^2/(\mu\Omega^2)$; it does not reverse the positive inertial correction. The position-coupled action by itself thus proves no stable quasi-static negative inertia.

A complementary reduction, derived in the Supplementary Information, uses a *velocity* coupling to a cyclic electromagnetic coordinate and is the mathematically appropriate starting point for a kinetic Schur complement (Fig. 1). The coefficients of these two models have different units unless the internal coordinates are independently normalized; their coupling constants cannot simply be equated.

Direct LC–flux response

The same device proposal separately specifies a capacitor charge Q, a physical inductance L_e, a capacitance C_e, and a local flux gradient $G = \partial_X \Phi$. For either winding sign $\sigma = \pm 1$, its conservative circuit coupling is

$$L_{\mathrm{LC}} = \frac{M}{2}\dot{X}^2 + \frac{L_e}{2}\dot{Q}^2 - \frac{Q^2}{2C_e} + \sigma G X \dot{Q} + FX. \tag{4}$$

Its Euler–Lagrange equations are $M\ddot{X} - \sigma G\dot{Q} = F$ and $L_e\ddot{Q} + Q/C_e = -\sigma G\dot{X}$. Exact charge elimination yields

$$\boxed{F(\omega) = -\omega^2 \left[M + \frac{C_e G^2}{1 - L_e C_e \omega^2} \right] X(\omega), \quad M_{\mathrm{LC}}(0) = M + C_e G^2.} \tag{5}$$

This increment is positive independent of winding orientation. Its SI units follow from $[C_e G^2] = \mathrm{kg}$; the input pair $C_e = 2.5\,\mu\mathrm{F}$ and $G = 0.22\,\mathrm{Wb\,m^{-1}}$ gives $C_e G^2 = 1.21 \times 10^{-7}\,\mathrm{kg}$, a source-model calculation rather than an observation. The LC and cyclic-channel responses cannot be identified through their shared symbol M alone; the explicit same-baseline obstruction is proved below in Eq. (11). A proposed device-side identification uses $\Lambda = L_b$ and $a = G_b$ for its cyclic channel; the two assignments as written violate SI dimensions by one time factor (Eq. (22)).

Whole-device translation check

Proposition 1 (Co-moving flux-gradient identity)**.** *If a rigid loop $\Gamma_X = \Gamma_0 + X$ and every source of its magnetic potential $\mathbf{A}_X(\mathbf{r}) = \mathbf{A}_0(\mathbf{r} - X)$ translate together in a translation-invariant environment, their gauge-invariant linked flux obeys*

$$\Phi(X) = \oint_{\Gamma_X} \mathbf{A}_X \cdot d\mathbf{l} = \oint_{\Gamma_0} \mathbf{A}_0 \cdot d\mathbf{l}, \qquad \partial_X \Phi = 0. \tag{6}$$

Proof. Substitute $\mathbf{r} = X + \mathbf{u}$ in the line integral; $d\mathbf{r} = d\mathbf{u}$. A smooth gauge change adds a closed-loop integral of a gradient, which vanishes. (QED)

Thus a nonzero flux gradient from internally co-moving coils must describe a *relative* coordinate, an anchored external magnetic source, or another explicitly non-co-moving reference. The derivative for rigid translation of the complete isolated assembly is zero under the theorem's conditions. This statement does not prohibit a physical relative-motion electromechanical coupling; it identifies the extra degrees of freedom and momentum bookkeeping required before calling X the total centre-of-mass coordinate.

Proposition 2 (Fixed-link flux rigidity in a field-free swept region)**.** *Let C_X be a smooth family of closed pickup paths while the magnetic source flux is fixed. If the ribbon swept between C_{X_1} and*

C_{X_2} *lies in a region where* $\mathbf{B} = 0$*, then*

$$\boxed{\Phi(C_{X_2}) - \Phi(C_{X_1}) = \int_{\Sigma_{12}} \mathbf{B} \cdot d\mathbf{S} = 0, \qquad \partial_X \Phi = 0.} \tag{7}$$

For an ideal toroidal flux tube and an exterior deformation preserving the linking number, the linked flux is $n\Phi_{\mathrm{core}}$ *and has no continuous geometry-dependent gradient.*

Proof. Apply Stokes' theorem to the oriented swept ribbon Σ_{12}; its boundary is $C_{X_2} - C_{X_1}$. The flux difference is the integral of $\nabla \times \mathbf{A} = \mathbf{B}$ over that ribbon. Since $\mathbf{B} = 0$ there, the difference vanishes. In an ideal confined-flux tube this is also the constancy of the integer linking number on smooth exterior deformations. (QED)

The proposed $G \sim N_h \Phi_{\mathrm{core}} a/h^2$ is therefore *not* a consequence of ideal fixed-link toroidal topology. A nonzero position derivative requires a measured nonideal/fringing-field flux, motion through a region with nonzero $\mathbf{B}$, variation of the source flux, or another specified field-dependent mechanism. In particular, a fixed-link, exterior, field-free pickup cannot obtain a nonzero position gradient merely from a nonzero vector potential. The Supplementary Information retains the design estimate but identifies this missing field-level derivation.

Exact one-channel reduction

A separate quadratic cyclic-channel ansatz is analysed without identifying its coefficients with those of Eq. (4). Let X denote the retained translation coordinate, ξ an internal harmonic mode, and η a cyclic linked-flux coordinate. Write

$$\begin{aligned} L_{\mathrm{micro}} = {} & \tfrac{1}{2} M \dot{X}^2 + \tfrac{1}{2} \mu \dot{\xi}^2 - \tfrac{1}{2} \kappa \xi^2 + \tfrac{1}{2} \Lambda \dot{\eta}^2 \\ & + a \dot{X} \dot{\eta} + b \xi \dot{\eta} + F X, \qquad M, \mu, \kappa, \Lambda > 0. \end{aligned} \tag{8}$$

Here a and b refer to coefficients in this action, not directly to unnormalized flux gradients. The conserved momentum $p_\eta = \Lambda \dot{\eta} + a \dot{X} + b \xi$ is fixed to zero on the declared channel. Elimination

of η by substitution (equivalently, Routh reduction at fixed zero momentum) gives

$$L_{\rm red} = \tfrac{1}{2}\left(M - \frac{a^2}{\Lambda}\right)\dot{X}^2 + \tfrac{1}{2}\mu\dot{\xi}^2 - \tfrac{1}{2}\left(\kappa + \frac{b^2}{\Lambda}\right)\xi^2 - \frac{ab}{\Lambda}\dot{X}\xi + FX. \tag{9}$$

For $\omega^2 \ll (\kappa + b^2/\Lambda)/\mu$, integrating out ξ yields the exact leading inertial coefficient

$$\boxed{M_{\rm eff}(0) = M - \frac{a^2}{\Lambda} + \frac{a^2 b^2/\Lambda^2}{\kappa + b^2/\Lambda} = M - \frac{a^2}{\Lambda}\frac{\kappa}{\kappa + b^2/\Lambda}.} \tag{10}$$

Proposition 3 (Nontrivial same-baseline matching obstruction)**.** *For $C_e, \Lambda, \kappa > 0$ and real G, a, b, suppose the physical LC action and the cyclic-channel action are asserted to describe the* same *translational coordinate with the* same *uncoupled kinetic mass M. Their zero-frequency response difference is exactly*

$$\boxed{M_{\rm LC}(0) - M_{\rm eff}(0) = C_e G^2 + \frac{a^2\kappa}{\Lambda\kappa + b^2} \geq 0,} \tag{11}$$

*with equality only when $G = a = 0$. **(QED)***

This is a restricted matching result, not a no-go theorem for all electromagnetic architectures. An additive physical interaction or a different bare-mass matching prescription would require its own explicit and dimensionally calibrated action; neither follows from a relabelling of the two displayed couplings.

No sign assumption beyond the written real quadratic action enters this identity. The associated frequency-dependent expression (without loss or feedback) is

$$M_{\rm eff}(\omega) = M - \frac{a^2}{\Lambda} + \frac{a^2 b^2/\Lambda^2}{\kappa + b^2/\Lambda - \mu\omega^2}, \tag{12}$$

valid away from the internal pole. Adding resistance or an external controller requires its own dissipative or driven dynamical model; a damping constant cannot be silently inserted into a conservative action.

Stability boundary

The X–η kinetic block of Eq. (8) is $K = \begin{pmatrix} M & a \\ a & \Lambda \end{pmatrix}$. A strictly positive quadratic kinetic energy requires $\Lambda > 0$ and $M\Lambda - a^2 > 0$. On the balanced one-channel branch $\kappa = b^2/\Lambda$. Substituting into Eq. (10) produces $M_{\text{eff}}(0) = M - a^2/(2\Lambda)$.

Proposition 4 (Balanced-branch lower bound)**.** *For the Lagrangian* (8) *with* $M, \mu, \Lambda, \kappa > 0$*,* $p_\eta = 0$*,* $\kappa = b^2/\Lambda$*, and a strictly positive kinetic matrix, the matched response satisfies*

$$\boxed{\frac{M_{\text{eff}}(0)}{M} = 1 - \frac{a^2}{2M\Lambda} > \frac{1}{2}.} \tag{13}$$

In particular, $M_{\text{eff}}(0) = 0$ *is incompatible with these assumptions.*

Proof. Positive definiteness gives $0 \le a^2/(M\Lambda) < 1$. The balanced condition reduces Eq. (10) to Eq. (13); the strict lower bound follows. (QED)

This conclusion is specific rather than universal. The same conservative quadratic class has a constructive non-balanced family that produces any strictly positive target ratio, at the cost of proximity to a singular kinetic matrix.

Proposition 5 (Strictly positive suppression family)**.** *For any prescribed* $0 < \delta < 1$*, there are positive* M, μ, Λ, κ *and real* a, b *with positive-definite kinetic energy and a stable internal harmonic mode such that the matched coefficient of Eq.* (10) *obeys*

$$\boxed{M_{\text{eff}}(0)/M = \delta.} \tag{14}$$

The endpoint $\delta = 0$ *is not obtained in this positive-definite family.*

Proof. Define $r = a^2/(M\Lambda)$ and $s = b^2/(\Lambda\kappa)$, both dimensionless. Equation (10) becomes $M_{\text{eff}}(0)/M = 1 - r/(1+s)$. Choose

$$r = 1 - \frac{\delta}{2}, \qquad s = \frac{\delta}{2(1-\delta)}. \tag{15}$$

Then $0 < r < 1$, $s > 0$, and $1 - r/(1+s) = \delta$ by exact substitution. Taking $a = \sqrt{M\Lambda r}$,

$b = \sqrt{\Lambda\kappa s}$ realizes the construction. Its kinetic principal determinant is $M\Lambda - a^2 = M\Lambda\delta/2 > 0$ for every $\delta > 0$, while it tends to zero as $\delta \downarrow 0$. Positivity of μ and κ supplies a positive internal oscillator energy. (QED)

The family establishes a tunable reduced-coordinate inertial coefficient within the assumed Lagrangian, not an engineering realization of the coefficients. Because the small-response limit approaches kinetic degeneracy, calibration uncertainty, neglected modes and electromagnetic backreaction become critical and require quantitative control. It also does not follow that the coordinate X equals the physical COM of an isolated entire apparatus. Establishing this requires deriving the coordinate definition and the momentum ledger for the complete device.

Recovered uniform-band existence theorem

The static positive-energy existence claim above admits a stronger finite-band form without adding a new device model. Put $\Omega_{\mathrm{red}}^2 = (\kappa + b^2/\Lambda)/\mu$ and $z = \omega^2/\Omega_{\mathrm{red}}^2$. In the strictly positive non-balanced family of Eq. (15), the *exact* expression (12) simplifies to

$$\boxed{\frac{M_{\mathrm{eff}}(\omega)}{M} = \delta\left(1 + \frac{z}{2(1-z)}\right), \quad 0 \le z < 1.} \tag{16}$$

Proposition 6 (Arbitrary finite-band suppression in the stipulated cyclic action)**.** *For every* $0 < \epsilon < 1$ *and* $0 < \beta < 1$*, positive model coefficients exist with a strictly positive-definite kinetic matrix and a stable internal mode such that*

$$\boxed{0 < M_{\mathrm{eff}}(\omega) \le \epsilon M \quad \textit{for every } \omega^2 \le \beta\Omega_{\mathrm{red}}^2.} \tag{17}$$

This is a uniform subresonant bound on the undamped reduced-coordinate response, not a complete-device Maxwell matching theorem.

Proof. Choose any $0 < \delta \le 2\epsilon(1-\beta)/(2-\beta)$ and use Eq. (15). With $r = a^2/(M\Lambda) = 1-\delta/2 < 1$, the kinetic determinant is $M\Lambda\delta/2 > 0$; $\kappa + b^2/\Lambda > 0$ ensures the internal mode is stable. Equation (16) is increasing in z on $[0, 1)$, hence its maximum for $z \le \beta$ is $\delta(2-\beta)/[2(1-\beta)] \le \epsilon$. (QED)

The formulation also includes an internal damping rate Γ as a calibration parameter. If the retained oscillator ξ has nonnegative Rayleigh damping $\mu\Gamma\dot{\xi}$ while the cyclic η-momentum remains conserved, replace z in Eq. (16) by the complex parameter

$$z_\Gamma = \frac{\omega^2 + i\Gamma\omega}{\Omega_{\rm red}^2}. \tag{18}$$

The same exact formula holds with z_Γ, and for any real frequency satisfying $|z_\Gamma| \le \beta < 1$ the triangle inequality gives

$$\boxed{\left|\frac{M_{\rm eff}(\omega)}{M}\right| \le \frac{\delta}{2}\left(1 + \frac{1}{1-\beta}\right) \le \epsilon.} \tag{19}$$

This conditional damped theorem is a statement about loss in the retained internal oscillator, not about a resistive or actively controlled eliminated η channel.

This proof also quantifies the cost of arbitrary suppression. Rescale the X–η kinetic block by its diagonal entries to $\left(\begin{smallmatrix} 1 & \sqrt{r} \\ \sqrt{r} & 1 \end{smallmatrix}\right)$. The corresponding spectral condition number is exactly

$$\boxed{\mathrm{cond}_2(K_{\rm normalized}) = \frac{1+\sqrt{1-\delta/2}}{1-\sqrt{1-\delta/2}} = \frac{2(1+\sqrt{1-\delta/2})^2}{\delta}.} \tag{20}$$

It diverges as $\delta \downarrow 0$ but is finite at every chosen positive δ. Thus the theorem proves existence at arbitrarily high yet nonzero suppression, while locating the precise lack of uniform robustness at the unattained zero endpoint.

Proposition 7 (Combined sector-level inertial and passive suppression)**.** *If,* in addition to the preceding positive-energy cyclic action*, its retained translation-sector monopole is covariantly completed by* $-M_{\rm eff}(0)c^2\int d\tau$ *with the specified background contributions treated separately, then*

$$m_{\rm inert}^{\rm COM,sector} = m_{\rm passive}^{\rm COM,sector} = \delta M \le \epsilon M. \tag{21}$$

Proof. At $\omega = 0$, Eq. (16) gives $M_{\rm eff}(0) = \delta M$. Expanding the stipulated proper-time monopole gives $+M_{\rm eff}(0)v^2/2 - M_{\rm eff}(0)\Phi$ at leading order. Both sector coefficients therefore coincide. This does not independently derive the proper-time completion of the full apparatus or the LC-to-cyclic matching. (QED)

Checking the original device-side identification

The microscopic cyclic-action reduction *does* prove the Schur reduction used above. The subsequent engineering corollary, however, *assumes* $\Lambda = L_b$ and $a = G_b$, identifying L_b as an inductance and $G_b = d\Phi_b/dX$ as a linked-flux gradient. Because $[\Lambda] = [L_b] = \mathrm{H}$ fixes $[\eta] = \mathrm{C}$ in $\Lambda\dot{\eta}^2/2$, dimensional homogeneity of $a\dot{X}\dot{\eta}$ requires

$$[a] = \frac{\mathrm{H\,C}}{\mathrm{m}} = \frac{\mathrm{Wb\,s}}{\mathrm{m}}, \qquad [G_b] = \frac{\mathrm{Wb}}{\mathrm{m}}. \tag{22}$$

Consequently $a = G_b$ as written lacks one factor with units of time. A canonical rescaling, if physically justified, must transform *both* Λ and a; their two equalities cannot be used unchanged. The required electromagnetic matching is not supplied by the algebraic Schur-reduction proof itself.

The formulation also separates a flat-space constant energy E_0 from the reduced kinetic coefficient. For a *single, isolated, co-moving* local monopole with no additional fields, reparametrization invariance fixes $S = -mc^2 \int d\tau$, whose flat expansion couples the constant $-mc^2$ and kinetic term $mv^2/2$. Therefore E_0 and $M_{\mathrm{eff}}(0)$ are not two independently adjustable monopole coefficients of that same complete isolated object. A background-supported or open-system split is possible, but its own gravitational and momentum terms then require matching. The sector-level equality (21) remains an exact theorem under its declared premise.

Circuit normalization and weak gravity

For a standard series LC circuit written using physical capacitor charge Q (coulombs), the conservative circuit Lagrangian is

$$L_{LC} = \tfrac{1}{2} L_e \dot{Q}^2 - \frac{Q^2}{2C_e}, \qquad \Omega_{LC}^2 = (L_e C_e)^{-1}. \tag{23}$$

The generalized inertia multiplying $\dot{Q}^2$ is L_e (henries); it is not $L_e C_e$, which has the dimension of seconds squared. If an alternative internal coordinate χ is chosen, its inertia and coupling

must be obtained by the same declared canonical rescaling. A linked-flux gradient $G = \partial\Phi/\partial X$ has units of weber per metre. Without a specified coordinate normalization, the identifications $\mu_k = LC$ in kilograms and $\Delta M_{\rm ct} = G^2/L_e$ in kilograms are not valid SI equalities. The latter has the dimensions of mass divided by time squared.

Even a correctly calibrated COM acceleration coefficient is not, by itself, a proof of suppressed gravitational weight or far-field source. General relativity couples to a complete conserved stress–energy tensor, including electromagnetic energy, mechanical stresses, supports and sources[4, 5]. An independently derived weak-field action and a gravitational measurement would be required to infer a passive weight coefficient, and a separate asymptotic or stress-energy calculation for active mass. The strongest *conditional* monopole-EFT statement of the source is retained: if the restricted COM translation sector admits the local covariant completion $-M_{\rm eff}(0)c^2\int d\tau$ with internal/background and finite-size contributions accounted for separately, its weak-field expansion contains both $\frac{1}{2}M_{\rm eff}(0)v^2$ and $-M_{\rm eff}(0)\Phi$. Thus the matched passive/inertial coefficient equality follows for that stipulated sector; identifying it with the measured passive weight of the complete apparatus still requires full-device matching.

Internal closure of the source's flux-gradient action

The selected 96-page manuscript itself gives the free LC action $L_e\dot{Q}^2/2 - Q^2/(2C_e)$ and the linked-flux term $GX\dot{Q}$, with $G = \partial\Phi/\partial X$. Retaining precisely these terms and the mechanical kinetic energy gives

$$L_{\rm LC+X} = \frac{1}{2}M\dot{X}^2 + \frac{1}{2}L_e\dot{Q}^2 - \frac{Q^2}{2C_e} + \sigma GX\dot{Q} + F_{\rm ext}X, \qquad \sigma \in \{-1, +1\}. \tag{24}$$

Changing flux orientation changes σ, but not the conclusion. The exact frequency-space force–displacement relation after eliminating Q is

$$F_{\rm ext}(\omega) = -\omega^2\left[M + \frac{C_eG^2}{1 - L_eC_e\omega^2}\right]X(\omega), \quad |\omega| < (L_eC_e)^{-1/2}. \tag{25}$$

Thus the source's directly declared LC–flux terms yield a strictly *positive* low-frequency inertia increment $\Delta M_{\mathrm{LC}} = C_e G^2$, not the negative counterterm required by the later matched-suppression law. At the source's illustrative values $C_e = 2.5\,\mu\mathrm{F}$ and $G = 0.22\,\mathrm{Wb\,m^{-1}}$, this contribution equals $1.21 \times 10^{-7}\,\mathrm{kg}$ (the baseline $M = 5\,\mathrm{kg}$). The numerical figure is a result of the declared quadratic circuit model, not a measurement of a physical apparatus.

The given energy formulation also expands $U_{\mathrm{mag}} = (L_e\dot{Q} + \Phi_0 + GX)^2/(2L_e)$: at second order that expression includes both the $GX\dot{Q}$ cross term and the separate static $G^2X^2/(2L_e)$ energy term. Omitting the latter is a truncation of the static stiffness ledger, not a new negative inertial mass term. Its force interpretation depends on complete magnetic-source co-energy and supports; the positive C_eG^2 coefficient of the explicitly specified velocity coupling is unchanged.

The original canonical normalization $Q = \sqrt{C_e}\,\chi$ assigns $[\chi] = \sqrt{\mathrm{J}}$ and $[L_eC_e] = \mathrm{s}^2$. Consequently its conversion $g_{\mathrm{pos}} = \Omega\sqrt{C_e}\,G$ has units $\sqrt{\mathrm{J}}/\mathrm{m}$ and $g_{\mathrm{pos}}^2/(L_eC_e\Omega^2)$ has units J/m^2, a *stiffness*, not kilograms. Replacing the drive frequency in $GX\dot{Q}$ with the resonant frequency Ω does not commute with the source's $\omega \to 0$ quasi-static limit. The later microscopic kinetic cross-term $a\dot{X}\dot{\eta}$ is a distinct term whose coefficient requires an independent dimensional and electrodynamic derivation from this same device.

Exact positive-energy window on the selected microscopic branch

For the source's cyclic-channel action, write $r = a^2/(M\Lambda)$ and $s = b^2/(\Lambda\kappa)$. At fixed cyclic momentum,

$$\frac{M_{\mathrm{eff}}(0)}{M} = 1 - \frac{r}{1+s} = \frac{1-r+s}{1+s}, \qquad 0 \le r < 1, \quad s \ge 0. \tag{26}$$

The microscopic positive-energy criterion is $M\Lambda - a^2 > 0$. On its explicitly chosen balanced condition $s = 1$, one obtains $M_{\mathrm{eff}}(0)/M = 1 - r/2 > 1/2$. More strongly, throughout the subresonant interval $0 \le \omega^2 < (\kappa + b^2/\Lambda)/\mu$,

$$M_{\mathrm{eff}}(\omega) = M - \frac{a^2}{\Lambda} + \frac{a^2b^2/\Lambda^2}{\kappa + b^2/\Lambda - \mu\omega^2} \ge M_{\mathrm{eff}}(0) > \frac{M}{2} \quad \text{(balanced)}. \tag{27}$$

For any proposed general-branch target $0 < M_{\mathrm{eff}}(0)/M \leq \delta < 1$, necessary conditions are $1 - r \leq \delta$ and $s \leq \delta/(1-\delta)$. They expose the joint near-degeneracy and stiffness-mismatch requirements rather than postulating a cancellation. The original objective of simultaneous quasi-static inertial and passive weak-field suppression is retained as the physical target; Eqs. (25)–(27) state what this paper's given actions establish without adding other papers or alternative apparatus models.

Experimental discriminants

An end-to-end test should independently determine: (i) the circuit capacitance, inductance, resistance and charge normalization; (ii) the flux profile and its gradient under actual body displacement; (iii) the multiport mechanical/electrical transfer functions for both drive-on and drive-off configurations; (iv) the cross-kinetic coefficients and normal-mode spectrum, including a positive-energy check; and (v) gravitational force and far-field observables independently of acceleration. The force-to-acceleration ratio must be evaluated after subtracting static stiffness and support reactions: otherwise a $1/\omega^2$ term from Eq. (2) can be misidentified as an inertia shift. The apparatus-specific numerical targets are design calculations rather than measurements.

At a predeclared low-frequency interval, a reproducible analysis would fit a force-to-displacement transfer function, then separate its constant, linear and quadratic frequency terms. The quadratic term determines the restricted inertial coefficient once geometry, gain, loss and boundary conditions are fixed. A measured small coefficient would remain an effective-response observation until complete-system momentum conservation and the gravity-side stress-energy ledger had been verified independently.

Discussion

The mechanism considered here is a Schur-complement correction in an electromechanical coordinate. Its exact sign depends on the complete retained quadratic form and on the constraints used during mode elimination; the position-coupled and velocity-coupled Lagrangians cannot be

treated interchangeably. The balanced branch has an unavoidable energy-positivity restriction and is not a stable route to zero coefficient within the stated microscopic model. A non-balanced positive-energy branch supplies the stronger exact uniform-band existence theorem, Eqs. (16)–(17), with an explicit conditioning cost (Eq. (20)). This is the full positive mathematical near-null result inside the proposed cyclic model; physical LC identification and complete-device gravitational matching are separately testable requirements. In addition, Eq. (7) rules out the claimed position gradient from ideal, fixed-link, field-free toroidal flux alone. No experimental measurements establishing a mass-null operating point are reported. Consequently, a compelling Nature submission requires at least one dimensionally calibrated full-device realization, a closed stability test, and measured transfer functions; gravitational claims require additional independent evidence. Technical derivations, device parameters, calibration protocols and extended discussions are collected in the Supplementary Information.

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Methods

Quadratic elimination. With $p_\eta = 0$, use $\dot{\eta} = -(a\dot{X} + b\xi)/\Lambda$ in Eq. (8). The corresponding Schur complement is Eq. (9). Eliminating ξ in frequency space and expanding around zero

frequency gives Eqs. (10) and (12). Detailed derivations, parameter tables, stress ansatz, calibration protocol and extensions are supplied in the Supplementary Information.

Stability audit. Sylvester's criterion applied to K gives $M > 0$, $\Lambda > 0$, and $M\Lambda - a^2 > 0$. The same restriction can be checked by the principal minors of the full kinetic matrix for $\dot{X}$, $\dot{\xi}$ and $\dot{\eta}$. No additional Hamiltonian assumption is introduced.

Flux-covariance and topological-rigidity checks. The fixed-link ribbon identity of Eq. (7) follows from Stokes' theorem in the swept field-free exterior. The global-translation derivative in Eq. (6) is evaluated with both the integration loop and the internally generated field translated by the same displacement. It differs from a derivative of the flux with respect to the separation of distinct parts.

Scope. The analysis uses both explicit actions already present in the device proposal and does not derive a dimensionally matched map between them. It does not assert a stable balanced zero-response realization or a measured passive weight suppression.

Data availability. All analytical inputs are given in the Supplementary Information. No experimental device data are reported.

Code availability. Standalone algebraic and SI-dimensional verification code is provided as `Supplementary_Code.txt` in the accompanying files. This code checks the specified analytic actions; no experimental device-acquisition code or measured mass-null data are reported.

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Author contributions

H.J.H. developed the theoretical analysis and drafted the manuscript.

Competing interests

The author declares no competing interests.

Additional information

Correspondence and requests for materials should be addressed to tobloqibs@icloud.com.

Supplementary Information is available for this paper.