Gaussian-T^3, Solid-Torus, and Möbius Observable Record Geometry and Effective Late-Time Selection on a Standard Quantum Background

Born-Conserving Dynamics, Likelihood Closure, and Terminal Constructive Witness

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Stable macroscopic records require both a rule for suppressing incompatible record coherences and a rule for identifying the alternatives that can become records. Standard quantum measurement theory supplies completely positive instruments and conditional trajectories, whereas stochastic state-reduction models add an autonomous reduction dynamics.[1–11] Here we show that a four-part record geometry—Gaussian structure, a marked three-torus, a solid-torus carrier and a Möbius side line—supports both routes in one mathematical framework. A dynamical Gaussian covariance on the full Hilbert space fixes a unique rank-one projector family and generates an autonomous positive stochastic process that converges almost surely to one projector with Born probabilities. An independent von Neumann Gaussian-probe construction gives exact discrete conditional reduction and converges to the same diffusive filtering law. The topology restricts the admissible record space and Möbius sector, while standard completely positive dilation theory preserves ensemble consistency.[12–15] The construction separates the stronger autonomous stochastic premise from the standard monitored-measurement realization and provides

explicit device and circuit completions in the Supplementary Information.

The quantum measurement problem combines two logically different questions. First, how does a quantum state produce a stable classical record that can be redundantly read? Second, if one demands an observer-independent single state in each individual realization, what physical dynamics chooses that state? Completely positive maps, instruments and dilation theory answer the first question operationally, and continuous-measurement theory describes conditional state trajectories.[1–7] They do not, by themselves, turn a closed-system unitary law into an autonomous stochastic reduction law. Conversely, dynamical-reduction theories introduce stochastic state evolution explicitly.[8–11] A useful theory should therefore state which part of the construction is standard quantum measurement physics and which part is a model-specific autonomous dynamics. Figure 1 summarizes the two routes and their shared record geometry.

We formulate this separation on a compact record carrier with four structural ingredients. The marked factor T^3 supplies periodic record coordinates. A solid torus $D_{\mathrm{bulk}} \simeq S^1 \times D^2$ supplies a retained bulk route, and a non-trivial real line bundle over its core provides the Möbius sign monodromy. The Gaussian component has two levels. At the observable level it defines positive-definite record kernels, using the classical fact that squared Hilbert distances generate positive Gaussian kernels.[12,13] At the autonomous level it is a positive, injective trace-class covariance operator with a physical Gaussian field and time scale. The latter is deliberately visible as a model primitive rather than being hidden in a downstream "decoherence" coefficient.

The complete derivations, device-level metrics, bosonic bath calculation, passive-circuit infrared closure, finite-record witnesses, historical intermediate formulations and all theorem proofs are retained without deletion in the Supplementary Information. The Article concentrates on the strongest common logical spine and the exact claim boundary.

The construction is designed to keep existence, dynamics and interpretation separate. Compact geometry is used to define and compare record configurations; complete positivity is established independently at the channel level; and the autonomous stochastic dynamics is introduced only when the stronger single-state claim is made. This separation is useful because several superficially similar statements—decoherence of an ensemble, conditional state update under monitoring, existence of a classical register, and autonomous objective reduction—are

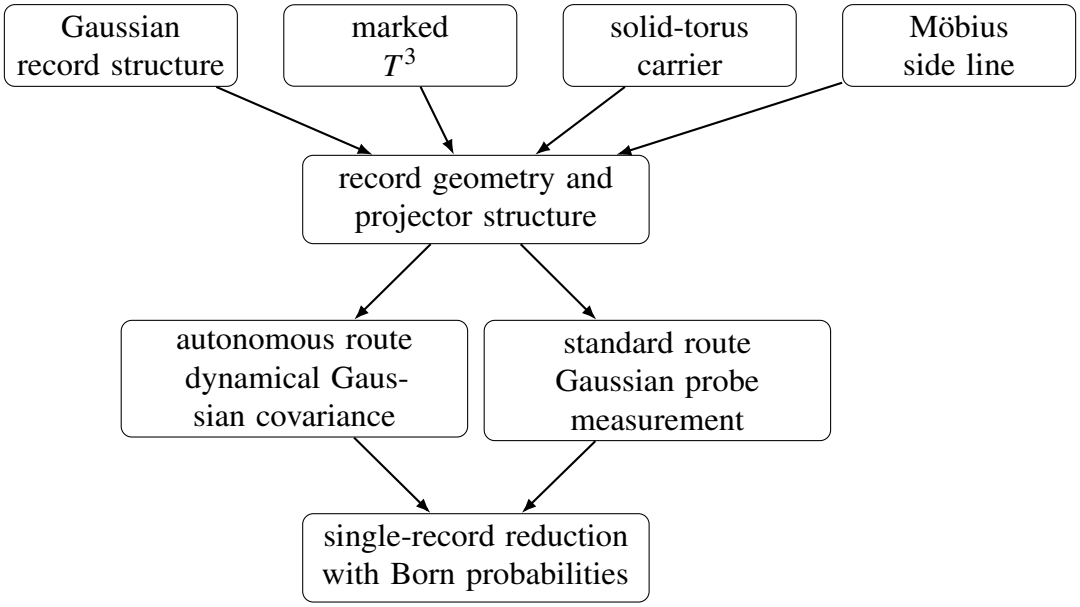


Figure 1: **Logical structure of the Article.** The four structural ingredients constrain the record space. The autonomous route uses an explicitly dynamical Gaussian covariance, whereas the standard route uses ordinary Gaussian probe measurements. Their physical premises remain distinct, but both yield single-record reduction with Born probabilities in their stated domains.

mathematically inequivalent. The long-form proof chain retains each of these layers rather than treating one as a synonym for another.

A second design principle is resolution independence. Early device theorems use finite coarse menus because real detectors have finite resolution, but a finite menu should not be mistaken for the fundamental state space or for a preferred basis chosen by hand. The final construction therefore places the dynamics on the full section Hilbert space and lets the simple spectrum of the Gaussian covariance select the terminal projectors. Finite menus reappear only as restrictions used to connect the abstract theorem to detector thresholds, likelihoods and error budgets. In this way the same framework contains both an infinite-dimensional model statement and finite laboratory observables without identifying the two.

Geometry fixes the record space

Let the final model Hilbert space be

$$\mathscr{H}_{\mathrm{full}} = L^2\left(T^3 \times D_{\mathrm{bulk}}; L_M^{\mathbb{C}}\right), \tag{1}$$

where L_M is the non-trivial real Möbius line bundle pulled back to the product carrier. The Möbius identification is a bundle condition, not a replacement of the solid-torus topology. In a core coordinate θ, a section changes sign after one primitive circuit, equivalently becoming anti-periodic on the connected orientation double cover. This distinction prevents an otherwise common category error in which the bulk manifold itself is treated as non-orientable.

At the coarse observable level, a Hilbert embedding Ψ defines

$$k(x,y) = \exp[-\|\Psi(x) - \Psi(y)\|^2]. \tag{2}$$

Schoenberg positivity and the Schur-product theorem imply that $\rho \mapsto K \odot \rho$ is positive on every finite record restriction, and unit diagonal makes it trace preserving.[12,13] A Gram representation gives an explicit Stinespring isometry, so this Gaussian coarse-graining is not merely a formal matrix operation.[2,3,15]

The same positivity statement also explains why the early finite-record theorems survive refinement: every finite principal submatrix of a positive Gaussian kernel is positive, and the corresponding Schur map remains a legitimate quantum channel. The later full-Hilbert construction does not use an arbitrary finite net to choose the terminal basis; instead, those finite channels are retained as experimentally accessible coarse restrictions of the full model.

The topological factors refine which histories can be represented as distinct records. The Möbius line has a connected two-fold orientation cover, and deck-paired histories can be encoded in orthogonal ancillary states. This supplies a gauge-invariant relative parity label without requiring an absolute global orientation of the side line. In the device realization, the same grading can be resolved by an ordinary system–bath coupling. A localized free-space electromagnetic resolving current is generically super-Ohmic in the infrared, whereas a semi-infinite matched transmission line supplies an explicit Ohmic completion; the full derivation and circuit rate are given in Supplementary Notes.

Gaussian covariance selects a basis

The autonomous route uses a stronger, explicitly dynamical Gaussian primitive. On $\mathscr{H}_{\mathrm{full}}$, let

$$C_G = \sum_{\alpha=1}^{\infty} \lambda_\alpha P_\alpha, \qquad \lambda_1 > \lambda_2 > \cdots > 0, \qquad \sum_\alpha \lambda_\alpha < \infty, \tag{3}$$

with rank-one projectors P_α, together with a physical rate $\gamma_G > 0$. The simple spectrum is essential: it removes the finite-net and basis-selection ambiguity that appears in purely coarse formulations. The collapse basis is the spectral resolution of C_G, hence is unique up to irrelevant eigenvector phases. This is a model statement about the Gaussian primitive; it is not claimed to follow from topology alone.

Set $g_\alpha = \gamma_G \lambda_\alpha$. Trace-class positivity implies $\sum_\alpha g_\alpha < \infty$, which controls the infinite stochastic sums. For independent real Gaussian coordinates $B_\alpha(t)$, define the unnormalised multiplicative propagator

$$M_t = \sum_\alpha \exp(\sqrt{g_\alpha} B_\alpha(t) - g_\alpha t) P_\alpha, \qquad \sigma_t = M_t \rho_0 M_t^\dagger. \tag{4}$$

The physical path measure is obtained by the normalized trace density, and $\rho_t = \sigma_t / \mathrm{Tr}\sigma_t$. This construction makes positivity manifest at every time and yields the nonlinear stochastic master equation by Itô and Girsanov calculus rather than by postulating that equation a second time.

Equivalently, the physical path law can be written as a Born-weighted mixture of Gaussian component measures. This representation is the reason the terminal law is not imposed separately: the same initial diagonal weights that appear in the density operator become the mixture weights of the mutually separating long-time path measures. It also exposes the model's strongest premise. If the dynamical Gaussian field is removed and only the static kernel is kept, the autonomous reduction theorem no longer follows; only the standard monitored-measurement route remains.

Almost-sure state reduction

Under the physical measure, the posterior diagonal weights $p_\alpha(t) = \mathrm{Tr}(P_\alpha \rho_t)$ are conditional probabilities for a countable latent mixture label. Brownian strong laws separate the path measures: under the α-component, $B_\alpha(t)/t \to 2\sqrt{g_\alpha}$, whereas the other normalized coordinates vanish. Martingale convergence then gives

$$p_\alpha(t) \longrightarrow \mathbf{1}_{\{I=\alpha\}} \quad \text{almost surely}, \tag{5}$$

and the fidelity–trace-distance inequality upgrades this to

$$\boxed{\|\rho_t - P_I\|_1 \longrightarrow 0 \quad \text{almost surely}}, \qquad \boxed{\Pr(I=\alpha) = \mathrm{Tr}(P_\alpha \rho_0)}. \tag{6}$$

Because the covariance is injective and trace class, its spectral projectors resolve the identity strongly on the entire model Hilbert space. No unmonitored tensor factor is left outside the theorem. The result is therefore stronger than the earlier finite-record martingale theorem retained in the Supplementary Information, while reducing to that theorem on every finite spectral truncation.

Thus the full model state, not only a finite coarse-record marginal, reduces to one covariance eigenprojector, with the Born terminal law. The ensemble remains linear:

$$\frac{d\bar{\rho}_t}{dt} = \sum_\alpha g_\alpha \mathscr{D}[P_\alpha]\bar{\rho}_t, \tag{7}$$

which is a norm-convergent GKSL generator.[16–19] The single-run stochastic reduction and linear completely positive ensemble are therefore compatible levels of description.

This coexistence is operationally important. Averaging over unresolved stochastic trajectories erases the random terminal label but does not invalidate the single-trajectory statement. Conversely, observing only the linear master equation is insufficient to infer a unique ontic unravelling. In the present model the unravelling is fixed by the dynamical Gaussian primitive itself; in the probe construction below it is fixed by the measured output record.

An arbitrary self-adjoint standard Hamiltonian can be restored by interaction-picture trans-

port. Writing $U_H(t) = e^{-iHt/\bar{h}}$ and $P_\alpha(t) = U_H(t)P_\alpha U_H^\dagger(t)$, one obtains the same almost-sure result in the co-moving basis,

$$\|\rho_t^H - U_H(t)P_I U_H^\dagger(t)\|_1 \to 0, \tag{8}$$

without a quantum-nondemolition commutator restriction. This statement is exact for the model defined above; the dynamical Gaussian covariance remains the non-unitary physical ingredient.

Standard probes recover the law

A separate route uses only ordinary monitored system–probe quantum mechanics. Let commuting record observables A_μ have distinct feature vectors a_i, couple them to fresh Gaussian pointer probes by a von Neumann translation interaction, and measure each probe coordinate. The exact Kraus density is Gaussian,

$$M_\delta(y) = (2\pi\delta)^{-m/4}\exp\left[-\frac{\|yI - 2\delta A\|^2}{4\delta}\right]. \tag{9}$$

For fixed $\delta > 0$, repeated readouts have the exact finite-mixture law

$$p(y_1,\ldots,y_n) = \sum_i p_i(0)\prod_{k=1}^n f_i(y_k). \tag{10}$$

Distinct features give $D_{\mathrm{KL}}(f_i\|f_j) = 2\delta\|a_i - a_j\|^2 > 0$. The strong law of large numbers therefore drives every wrong likelihood ratio to zero, giving exact conditional trace-norm reduction to one record projector and the same Born probabilities. This is a standard measurement result, not an autonomous collapse of an unmonitored closed system.[1,4–7]

Taking the diffusive limit yields the quantum-filtering equation

$$d\rho_t = \sum_\mu \mathscr{D}[A_\mu]\rho_t\,dt + \sum_\mu \mathscr{H}[A_\mu]\rho_t\,dW_\mu(t), \tag{11}$$

with the usual innovations process.[6,7] On the finite record sector, this equation has the same path law as the autonomous Hilbert–Wiener realization when their coefficients and initial conditions

are matched. The physical interpretations are nevertheless different: in the probe route W_t is the innovation associated with an observed output; in the autonomous route the Gaussian field is part of the model dynamics. The equality of stochastic equations does not erase that distinction.

The exact fixed-step theorem is also useful because it does not rely on the diffusion approximation. For any positive probe step, pairwise Kullback–Leibler separation makes the Bayesian posterior consistent, so the monitored conditional state converges before any continuum limit is taken. The stochastic differential equation is therefore a controlled weak-measurement limit of an already well-defined discrete instrument rather than a formal continuum ansatz.

Topology and device realization

The long-form construction shows how the abstract record metric can be realized from time-averaged detector current, environment imprint and visibility variables. It also separates two different uses of topology. First, the topology determines the admissible section space through T^3, D_{bulk} and L_M. Second, a parity-sensitive device can resolve the Möbius history grading through an ordinary environment channel. These roles should not be conflated: the full-Hilbert autonomous reduction uses the topology through the Hilbert space and covariance spectrum, whereas a separate parity readout is an operational measurement channel on that already-defined space.

For an explicit passive circuit, a semi-infinite transmission line with characteristic impedance Z_0, port-charge contrast Δq_{p} and form factor $F(\omega)$ produces

$$J_M^{\mathrm{TL}}(\omega) = \alpha_{\mathrm{TL}}\,\omega|F(\omega)|^2, \qquad \alpha_{\mathrm{TL}} = \frac{(\Delta q_{\mathrm{p}})^2 Z_0}{4\pi\bar{h}}. \tag{12}$$

At positive temperature the long-time dephasing exponent is linear and gives the standard Johnson–Nyquist scaling; at zero temperature the decay is algebraic.[20–23] These device results are not required to define the autonomous Gaussian covariance primitive, but they provide a standard-physics realization for a topology-sensitive record channel and a quantitative experimental interface.

The device analysis also provides a useful negative control. A smooth localized conserved

current coupled only to free-space electromagnetic modes is super-Ohmic at low frequency and therefore does not generically produce complete long-time parity projection. The matched line supplies the missing low-frequency continuum explicitly. This distinction prevents a topological sign from being mistaken for a universal dephasing rate and identifies which part of the observable selection can, in principle, be altered by circuit engineering.

Scope and implications

The framework makes three claims at different logical strengths. First, the four geometric ingredients admit positive Gaussian record channels, exact classical registers and stable redundant readout within standard quantum information theory. Second, when the Gaussian ingredient is dynamical in the explicit trace-class stochastic sense above, the full model Hilbert state undergoes autonomous almost-sure rank-one reduction with Born probabilities. Third, monitored Gaussian probes provide an independent standard-physics conditional-reduction route and reproduce the same diffusive law on their common finite record sector.

These claims do not imply that topology alone modifies the Schrödinger equation, nor that standard closed-system unitary evolution secretly contains stochastic non-unitarity. The dynamical Gaussian covariance is the explicit model-specific physical content responsible for the autonomous route. That transparency is important because stochastic reduction models are known to require dynamics beyond ordinary unitary evolution, even when their ensemble equations are completely positive.[8–11] Conversely, the probe theorem does not need such an autonomous premise and can be read entirely within standard measurement and filtering theory. The shared geometry makes the relation between these two routes mathematically explicit rather than rhetorically merging them.

The resulting hierarchy is directly testable at the level appropriate to each route. The monitored route predicts ordinary measurement statistics and circuit-dependent coherence decay already accessible to standard detector characterization. The autonomous route makes the stronger statement that the stochastic Gaussian field is physical even without an external observer; its rate spectrum is therefore model data, not a quantity inferred merely from the

existence of the Möbius bundle. Future comparison with experiment would have to constrain that spectrum independently of the standard measurement back-action channel.

A further consequence is methodological. The same ensemble master equation can possess inequivalent stochastic unravellings, so agreement at the level of a density-operator semigroup does not by itself establish an observer-independent collapse mechanism. In the present construction that ambiguity is resolved only because the autonomous path measure is part of the Gaussian primitive, whereas the monitored path measure is fixed by the actually observed probe record. This distinction supplies a concrete criterion for comparing future implementations: measurements of unconditional decoherence constrain the ensemble generator, while trajectory-resolved detector statistics constrain the monitored unravelling, and any proposed autonomous reduction must additionally specify the physical stochastic law that selects its trajectory measure.

The topology enters this comparison indirectly but non-trivially. It fixes the domain on which the covariance operator acts and therefore can reshape the admissible spectrum and projectors, while it does not by itself determine the stochastic intensity. This separation allows topology-sensitive devices to test record geometry without assuming that every topological label automatically produces collapse. Conversely, if an autonomous Gaussian spectrum is postulated, the full-Hilbert theorem shows precisely how its eigenprojectors and rates determine the terminal alternatives and the characteristic reduction times.

The Supplementary Information retains the full 99-page integrated technical development, including the universal Stinespring architecture, all intermediate sufficient-condition theorems and their discharge, detector embeddings, multi-basin theory, Möbius double-cover construction, exact oscillator-bath influence functional, infrared classification, passive-circuit completion, terminal finite-outcome witness, full-Hilbert proof and detailed error budget.

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Table 1 | Logical status of the principal results

Layer	Input	Proven output
Record geometry	Gaussian Hilbert geometry, T^3, solid torus, Möbius line	Positive record kernel, admissible topological sector, Stinespring realization
Autonomous reduction	Dynamical trace-class Gaussian covariance with physical Gaussian field	Full-Hilbert almost-sure rank-one reduction and Born terminal law
Standard probe	von Neumann coupling to fresh Gaussian probes and observed pointer outputs	Exact fixed-step conditional reduction, Born probabilities and diffusive filtering limit
Device interface	detector readout variables or parity-sensitive passive circuit	Explicit distances, error bounds, bath spectral density and topology-sensitive dephasing rates

Methods

Four structural ingredients

The final autonomous model uses four model-specific structural ingredients. The first is a marked three-torus $T^3 = (\mathbb{R}/2\pi\mathbb{Z})^3$ with normalized Haar measure. The second is a standard solid torus $D_{\text{bulk}} = S^1 \times D^2$. The third is the non-trivial real Möbius line bundle $L_M \to S^1$, pulled back to $T^3 \times D_{\text{bulk}}$ and complexified. The fourth is the dynamical Gaussian covariance described below. Standard Hilbert-space quantum mechanics, self-adjoint Hamiltonian evolution, completely positive maps, Itô/Girsanov calculus and ordinary monitored-probe theory are treated as background mathematics and physics, not as additional model-specific geometric ingredients.

Full Hilbert space

The model space is $\mathscr{H}_{\text{full}} = L^2(T^3 \times D_{\text{bulk}}; L_M^{\mathbb{C}})$. The Möbius structure is imposed as a bundle/section condition rather than as a second dynamical semigroup. This choice removes an ambiguity that arises if one simultaneously interprets the same parity structure as an exact static cancellation and as a finite-time exponential channel. A separate parity-sensitive detector remains possible as an ordinary measurement channel on $\mathscr{H}_{\text{full}}$.

Dynamical Gaussian covariance

The Gaussian primitive is a positive injective trace-class operator C_G with simple spectrum $\{\lambda_\alpha, P_\alpha\}$, a positive rate γ_G, and independent centered standard Gaussian coordinates. The physical rates are $g_\alpha = \gamma_G \lambda_\alpha$. Simple spectrum fixes the rank-one projector resolution without an auxiliary ε-net or finite-menu selector, and trace class gives $\sum_\alpha g_\alpha < \infty$. The positive multiplicative propagator and its normalized trace density define the physical path measure. The normalized stochastic master equation follows from Itô normalization and Girsanov transformation; it is not separately inserted as an independent fifth premise.

Born terminal law

The physical path measure is a countable mixture of components labelled by α with initial weights $\mathrm{Tr}(P_\alpha \rho_0)$. Strong-law separation of the Gaussian coordinates makes the terminal label measurable, and posterior martingales converge to its indicator. Trace-norm convergence follows from the fidelity bound. This proves both almost-sure rank-one reduction and the Born terminal law on the full model Hilbert space. Detailed measure-theoretic steps are in Supplementary Notes.

Standard Gaussian probes

For the independent monitored route, a system observable vector $A = (A_1, \ldots, A_m)$ is coupled to fresh Gaussian pointer degrees of freedom by a von Neumann translation unitary. Reading the pointer gives the Gaussian Kraus density written in the main text. At fixed step δ, the likelihoods associated with distinct feature vectors have strictly positive Kullback–Leibler divergence; repeated readout therefore gives almost-sure posterior concentration. In the weak-step limit the output process converges to the usual continuous-measurement innovations equation and the conditioned density operator obeys the stated diffusive filtering equation.[1,4–7]

Observable and circuit layers

The Supplementary Information contains the full device embedding based on time-averaged current, environment imprint and visibility coordinates; Gaussian pointer likelihoods; no-return and first-passage bounds; a block-preserving GKSL witness; the Möbius orientation-cover encoding; exact bosonic influence functional; free-space infrared classification; and semi-infinite matched-line completion. Standard open-system and circuit formulas follow Refs. 2–10. These constructions are used as explicit realizations or experimental interfaces and are not silently promoted to additional autonomous primitives.

Audit and theorem provenance

The technical supplement retains every substantive derivation from the integrated long-form manuscript. Earlier sufficient-condition statements are labelled as intermediate conditions and are explicitly discharged by terminal witnesses rather than counted as additional foundational assumptions. Earlier static and finite-record Gaussian formulations are retained as intermediate realizations; the final foundational primitive list is the four-part full-Hilbert formulation used in the principal theorem. Duplicate bibliographic entries have been removed. The two reduction routes are kept logically separate throughout: monitored filtering is standard measurement physics, while autonomous reduction depends on the explicitly dynamical Gaussian covariance primitive.

AI-assisted drafting and verification

Generative AI tools were used interactively during manuscript drafting, equation checking, consistency auditing and submission-format preparation. The author reviewed the resulting text, mathematical statements, citations and files and takes responsibility for the final manuscript and its claims.

Data availability

No new experimental or observational dataset is reported. All quantities required to evaluate the analytic statements are defined in the Article and Supplementary Information.

Code availability

No custom numerical code is required for the analytic theorems. The TeX sources and audit records are included in the submission package to permit exact reconstruction of the manuscript files.

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Author contributions

Jaejun Han conceived the framework, developed the mathematical construction and prepared the manuscript.

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Additional information

Supplementary Information is available for this paper.

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